

***Project 4.4 Wolves and moose**

For this project, we assume that you have read Section 4.4.

Predator-prey models are commonly used in biology because they can model a wide range of ecological relationships, e.g., wolves and moose, koala and eucalyptus, or humans and tuna. In the simplest incarnation, the livelihood of a population of predators is dependent solely on the availability of a population of prey. The population of prey, in turn, is kept in check by the predators.

In the 1920's, Alfred Lotka and Vito Volterra independently introduced the now-famous Lotka-Volterra equations to model predator-prey relationships. The model consists of a pair of related differential equations that describe the sizes of the two populations over time. We will approximate the differential equations with discrete difference equations. Let's assume that the predators are wolves and the prey are moose. We will represent the sizes of the moose and wolf populations at the end of month t with $M(t)$ and $W(t)$, respectively. The difference equations describing the populations of wolves and moose are:

$$M(t) = M(t - \Delta t) + b_M M(t - \Delta t) \Delta t - d_M W(t - \Delta t) M(t - \Delta t) \Delta t$$

$$W(t) = W(t - \Delta t) + b_W W(t - \Delta t) M(t - \Delta t) \Delta t - d_W W(t - \Delta t) \Delta t$$

where

- b_M is the moose birth rate (per month)
- d_M is the moose death rate, or the rate at which a wolf kills a moose that it encounters (per month)
- b_W is the wolf birth rate, or the moose death rate \times how efficiently an eaten moose produces a new wolf (per month)
- d_W is the wolf death rate (per month)

Let's look at these equations more closely. In the first equation, the term $b_M M(t - \Delta t) \Delta t$ represents the net number of moose births in the last time step, in the absence of wolves, and the term $d_M W(t - \Delta t) M(t - \Delta t) \Delta t$ represents the number of moose deaths in the last time step. Notice that this term is dependent on both the number of wolves and the number of moose: $W(t - \Delta t) M(t - \Delta t)$ is the number of possible wolf-moose encounters and $d_M \Delta t$ is the rate at which a wolf kills a moose that it encounters in a time step of length Δt .

In the second equation, the term $b_W W(t - \Delta t) M(t - \Delta t) \Delta t$ represents the number of wolf births per month. Notice that this number is also proportional to both the number of wolves and the number of moose, the idea being that wolves will give birth to more offspring when food is plentiful. As described above, b_W is actually based on two quantities, the moose death rate (since wolves have to eat moose to thrive and have offspring) and how efficiently a wolf can use the energy gained by eating a moose to give birth to a new wolf. The term $d_W W(t - \Delta t) \Delta t$ represents the net number of wolf deaths per month in the absence of moose.

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In this project, you will write a program that uses these difference equations to model the dynamic sizes of a population of wolves and a population of moose over time. There are three parts to the project. In the first part, you will use the Lotka-Volterra model to simulate a baseline scenario. In the second part, you will model the effects that hunting the wolves have on the populations. And, in the third part, you will create a more realistic simulation in which the sizes of the populations are limited by the natural resources available in the area.

Part 1: Implement the Lotka-Volterra model

Write a function in Python

```
PP(preypop, predpop, dt, months)
```

that simulates this predator prey model using the difference equations above. The parameters `preypop` and `predpop` are the initial sizes of the prey and predator populations ($M(0)$ and $W(0)$), respectively, `dt` (Δt) is the time interval used in the simulation, and `months` is the number of months (maximum value of t) for which to run the simulation. To cut back on the number of parameters, you can assign constant birth and death rates to local variables inside your function. Start by trying

```
birthRateMoose = 0.5      #  $b_M$ 
deathRateMoose = 0.02    #  $d_M$ 
birthRateWolves = 0.005  #  $b_W = d_M \times \text{efficiency } 0.25$ 
deathRateWolves = 0.75   #  $d_W$ 
```

Your function should plot, using `matplotlib`, the sizes of the wolf and moose populations over time, as the simulation progresses. Write a program that calls your `PP` function to simulate 500 moose and 25 wolves for 5 years with `dt = 0.01`.

Question 4.4.1 *What happens to the sizes of the populations over time? Why do these changes occur?*

Part 2: Here come the hunters!

Now suppose the wolves begin to threaten local ranchers' livestock (in Wyoming, for example) and the ranchers begin killing the wolves. Simulate this effect by increasing the wolf death rate d_W .

Question 4.4.2 *What is the effect on the moose population?*

Question 4.4.3 *What would the wolf death rate need to be for the wolf population to die out within five years? Note that the death rate can exceed 1. Try increasing the value of d_W slowly and watch what happens. If it seems like you can never kill all the wolves, read on.*

Killing off the wolves appears to be impossible because the equations you are using will never let the value reach zero. (Why?) To compensate, we can set either population to zero when it falls below some threshold, say 1.0. (After all, you can't really have a fraction of a wolf.) To do this, insert the following statements into the body of your `for` loop after you increment the predator and prey populations, and try answering the previous question again.

```

if preyPop < 1.0:
    preyPop = 0.0
if predPop < 1.0:
    predPop = 0.0

```

(Replace `preyPop` and `predPop` with the names you use for the current sizes of the populations.) As we will see shortly, the first two statements will assign 0 to `preyPop` if it is less than 1. The second two statements do the same for `predPop`.

Part 3: Modeling constrained growth

In the simulation so far, we have assumed that a population can grow without bound. For example, if the wolf population died out, the moose population would grow exponentially. In reality, an ecosystem can only support a limited size population due to constraints on space, food, etc. This limit is known as a *carrying capacity*. We can model the moose carrying capacity in a simple way by decreasing the moose birth rate proportionally to the size of the moose population. Specifically, let MCC represent the moose population carrying capacity, which is the maximum number of moose the ecosystem can support. Then change the moose birth term $b_M M(t - \Delta t)\Delta t$ to

$$b_M (1 - M(t - \Delta t)/MCC) M(t - \Delta t)\Delta t.$$

Notice that now, as the moose population size approaches the carrying capacity, the birth rate slows.

Question 4.4.4 *Why does this change cause the moose birth rate to slow as the size of the moose population approaches the carrying capacity?*

Implement this change to your simulation, setting the moose carrying capacity to 750, and run it again with the original birth and death rates, with 500 moose and 25 wolves, for 10 years.

Question 4.4.5 *How does the result differ from your previous run? What does the result demonstrate? Does the moose population reach its carrying capacity of 750? If not, what birth and/or death rate parameters would need to change to allow this to happen?*

Question 4.4.6 *Reinstate the original birth and death rates, and introduce hunting again; now what would the wolf death rate need to be for the wolf population to die out within five years?*