*Project 4.3 Market penetration

For this project, we assume that you have read Section 4.4.

In this project, you will write a program that simulates the adoption of a new product in a market over time (i.e., the product's "market penetration") using a difference equation developed by marketing researcher Frank Bass in 1969 [5]. The *Bass diffusion model* assumes that there is one product and a fixed population of N eventual adopters of that product. In the difference equation, $A(t)$ is the fraction (between 0 and 1) of the population that has adopted the new product t weeks after the product launch. Like the difference equations in Section 4.4, the value of $A(t)$ will depend on the value of $A(t - \Delta t)$, the fraction of the population that had adopted the product at the previous time step, $t - \Delta t$. Since t is measured in weeks, Δt is some fraction of a week. The rate of product adoption depends on two factors.

1. The segment of the population that has not yet adopted the product adopts it at a constant adoption rate of r due to chance alone. By the definition of $A(t)$, the fraction of the population that *had* adopted the product at the previous time step is $A(t - \Delta t)$. Therefore, the fraction of the population that *had not yet* adopted the product at the previous time step is $1 - A(t - \Delta t)$. So the fraction of new adopters during week t from this constant adoption rate is

$$r \cdot (1 - A(t - \Delta t)) \cdot \Delta t.$$

2. Second, the rate of adoption is affected by word of mouth within the population. Members of the population who have already adopted the product can influence those who have not. The more adopters there are, the more potential interactions exist between adopters and non-adopters, which boosts the adoption rate. The fraction of all potential interactions that are between adopters and non-adopters in the previous time step is

$$\underbrace{A(t - \Delta t)}_{\substack{\text{fraction that}\\ \text{are adopters}}} \cdot \underbrace{(1 - A(t - \Delta t))}_{\substack{\text{fraction that}\\ \text{are not adopters}}}.$$

The fraction of these interactions that result in a new adoption during one week is called the *social contagion*. We will denote the social contagion by s. So the fraction of new adopters due to social contagion during the time step ending at week t is

$$s \cdot A(t - \Delta t) \cdot (1 - A(t - \Delta t)) \cdot \Delta t.$$

The social contagion measures how successfully adopters are able to convince non-adopters that they should adopt the product. At the extremes, if $s = 1$, then every interaction between a non-adopter and an adopter results in the non-adopter adopting the product. On the other hand, if $s = 0$, then the current adopters cannot convince any non-adopters to adopt the product.

Putting these two parts together, the difference equation for the Bass diffusion model is

$$A(t) = A(t - \Delta t) + \underbrace{r \cdot (1 - A(t - \Delta t)) \cdot \Delta t}_{\substack{\text{fraction of new adopters} \\ \text{from constant rate}}} + \underbrace{s \cdot A(t - \Delta t) \cdot (1 - A(t - \Delta t)) \cdot \Delta t}_{\substack{\text{fraction of new adopters} \\ \text{from social contagion}}}$$

Part 1: Implement the Bass diffusion model

To implement the Bass diffusion model, write a function

```
productDiffusion(chanceAdoption, socialContagion, weeks, dt)
```

The parameters `chanceAdoption` and `socialContagion` are the values of r and s, respectively. The last two parameters give the number of `weeks` to simulate and the value of Δt to use. Your function should plot two curves, both with time on the x-axis and the proportion of adopters on the y-axis. The first curve is the total fraction of the population that has adopted the product by time t. This is $A(t)$ in the difference equation above. The second curve will be the rate of change of $A(t)$. You can calculate this by the formula

$$\frac{A(t) - A(t - \Delta t)}{\Delta t}.$$

Equivalently, this rate of change can be thought of as the fraction of new adopters at any time step, normalized to a weekly rate, i.e., the fraction of the population added in that time step divided by `dt`.

Write a program that uses your function to simulate a product launch over 15 weeks, using $\Delta t = 0.01$. For this product launch, we will expect that adoption of the product will move slowly without a social effect, but that social contagion will have a significant impact. To model these assumptions, use $r = 0.002$ and $s = 1.03$.

Question 4.3.1 *Describe the picture and explain the pattern of new adoptions and the resulting pattern of total adoptions over the 15-week launch.*

Question 4.3.2 *Now make r very small but leave s the same ($r = 0.00001$, $s = 1.03$), and answer the same question. What kind of market does this represent?*

Question 4.3.3 *Now set r to be 100 times its original value and s to be zero ($r = 0.2$, $s = 0$), and answer the first question again. What kind of market does this represent?*

Part 2: Influentials and imitators

In a real marketplace, some adopters are more influential than others. Therefore, to be more realistic, we will now partition the entire population into two groups called *influentials* and *imitators* [65]. Influentials are only influenced by other influentials, while imitators can be influenced by either group. The numbers of influentials and imitators in the population are N_A and N_B, respectively, so the total population size is $N = N_A + N_B$. We will let $A(t)$ now represent the fraction of the influential population that has adopted the product at time t and let $B(t)$ represent the fraction of the imitator population that has adopted the product at time t.

The adoption rate of the influentials follows the same difference equation as before, except that we will denote the adoption rate and social contagion for the influentials with r_A and s_A.

$$A(t) = A(t - \Delta t) + r_A \cdot (1 - A(t - \Delta t)) \cdot \Delta t + s_A \cdot A(t - \Delta t) \cdot (1 - A(t - \Delta t)) \cdot \Delta t$$

The adoption rate of the imitators will be different because they value the opinions of both influentials and other imitators. Let r_B and s_B represent the adoption rate and social contagion for the imitators. Another parameter, w (between 0 and 1), will indicate how much the imitators value the opinions of the influentials over the other imitators. At the extremes, $w = 1$ means that the imitators are influenced heavily by influentials and not at all by other imitators. On the other hand, $w = 0$ means that they are not at all influenced by influentials, but are influenced by imitators. We will break the difference equation for $B(t)$ into three parts.

1. First, there is a constant rate of adoptions from among the imitators that have not yet adopted, just like the first part of the difference equation for $A(t)$:

$$r_B \cdot (1 - B(t - \Delta t)) \cdot \Delta t$$

2. Second, there is a fraction of the imitators who have not yet adopted who will be influenced to adopt, through social contagion, by influential adopters.

$$w \cdot s_B \cdot \underbrace{A(t - \Delta t)}_{\substack{\text{fraction of} \\ \text{influentials who} \\ \text{have adopted}}} \cdot \underbrace{(1 - B(t - \Delta t))}_{\substack{\text{fraction of} \\ \text{imitators who} \\ \text{have not adopted}}} \cdot \Delta t.$$

   Recall from above that w is the extent to which imitators are more likely to be influenced by influentials than other imitators.

3. Third, there is a fraction of the imitators who have not yet adopted who will be influenced to adopt, through social contagion, by other imitators who have already adopted.

$$(1 - w) \cdot s_B \cdot \underbrace{B(t - \Delta t)}_{\substack{\text{fraction of} \\ \text{imitators who} \\ \text{have adopted}}} \cdot \underbrace{(1 - B(t - \Delta t))}_{\substack{\text{fraction of} \\ \text{imitators who} \\ \text{have not adopted}}} \cdot \Delta t.$$

   The term $1 - w$ is the extent to which imitators are likely to be influenced by other imitators, compared to influentials.

Putting these three parts together, we have the difference equation modeling the growth of the fraction of imitators who adopt the product.

$$\begin{aligned} B(t) = B(t-\Delta t) &+ r_B \cdot (1 - B(t-\Delta t)) \cdot \Delta t \\ &+ w \cdot s_B \cdot A(t-\Delta t) \cdot (1 - B(t-\Delta t)) \cdot \Delta t \\ &+ (1-w) \cdot s_B \cdot B(t-\Delta t) \cdot (1 - B(t-\Delta t)) \cdot \Delta t \end{aligned}$$

Now write a function

```
productDiffusion2(inSize, imSize, rIn, sIn, rIm, sIm, weight, weeks, dt)
```

that implements this product diffusion model with influentials and imitators. The parameters are similar to the previous function (but their names have been shortened). The first two parameters are the sizes of the influential and imitator populations, respectively. The third and fourth parameters are the adoption rate (r_A) and social contagion (s_A) for the influentials, respectively. The fifth and sixth parameters are the same values (r_B and s_B) for the imitators. The seventh parameter `weight` is the value of w. Your function should produce two plots. In the first, plot the new adoptions for each group, and the total rate of new adoptions, over time (as before, normalized by dividing by `dt`). In the second, plot the total adoptions for each group, and the total adoptions for both groups together, over time. Unlike in the previous function, plot the *numbers* of adopters in each group rather than the fractions of adopters, so that the different sizes of each population are taken into account. (To do this, just multiply the fraction by the total size of the appropriate population.)

Write a program that uses your function to simulate the same product launch as before, except now there are 600 influentials and 400 imitators in a total population of 1000. The adoption rate and social contagion for the influentials are the same as before ($r_A = 0.002$ and $s_A = 1.03$), but these values are $r_B = 0$ and $s_B = 0.8$ for the imitators. Use a value of $w = 0.6$, meaning that the imitators value the opinions of the influentials over other imitators.

Question 4.3.4 *Describe the picture and explain the pattern of new adoptions and the resulting pattern of total adoptions. Point out any patterns that you find interesting.*

Question 4.3.5 *Now set $w = 0.01$ and rerun the simulation. Describe the new picture and explain how and why this changes the results.*