4.8 PROJECTS

Project 4.1 Parasitic relationships

For this project, we assume that you have read the material on discrete difference equations in Section 4.4.

A parasite is an organism that lives either on or inside another organism for part of its life. A parasitoid is a parasitic organism that eventually kills its host. A parasitoid insect infects a host insect by laying eggs inside it then, when these eggs later hatch into larvae, they feed on the live host. (Cool, huh?) When the host eventually dies, the parasitoid adults emerge from the host body.

The *Nicholson-Bailey model*, first proposed by Alexander Nicholson and Victor Bailey in 1935 [43], is a pair of difference equations that attempt to simulate the relative population sizes of parasitoids and their hosts. We represent the size of the host population in year t with $H(t)$ and the size of the parasitoid population in year t with $P(t)$. Then the difference equations describing this model are

$$H(t) = r \cdot H(t-1) \cdot e^{-aP(t-1)}$$

$$P(t) = c \cdot H(t-1) \cdot \left(1 - e^{-aP(t-1)}\right)$$

where

- r is the average number of surviving offspring from an uninfected host,
- c is the average number of eggs that hatch inside a single host, and
- a is a scaling factor describing the searching efficiency or search area of the parasitoids (higher is more efficient).

The value $(1 - e^{-aP(t-1)})$ is the probability that a host is infected when there are $P(t-1)$ parasitoids, where e is Euler's number (the base of the natural logarithm). Therefore,

$$H(t-1) \cdot \left(1 - e^{-aP(t-1)}\right)$$

is the number of hosts that are infected during year $t - 1$. Multiplying this by c gives us $P(t)$, the number of new parasitoids hatching in year t. Notice that the probability of infection grows exponentially as the size of the parasitoid population grows. A higher value of a also increases the probability of infection.

Question 4.1.1 *Similarly explain the meaning of the difference equation for $H(t)$. ($e^{-aP(t-1)}$ is the probability that a host is not infected.)*

Part 1: Implement the model

To implement this model, write a function

```
NB(hostPop, paraPop, r, c, a, years)
```

that uses these difference equations to plot both population sizes over time. Your function should plot these values in two different ways (resulting in two different plots). First, plot the host population size on the x-axis and the parasitoid population size on the y-axis. So each point represents the two population sizes in a particular year. Second, plot both population sizes on the y-axis, with time on the x-axis. To show both population sizes on the same plot, call the `pyplot.plot` function for each population list before calling `pyplot.show`. To label each line and include a legend, see the end of Section 4.2.

Question 4.1.2 *Write a* `main` *function that calls your* `NB` *function to simulate initial populations of 24 hosts and 12 parasitoids for 35 years. Use values of* $r = 2$, $c = 1$, *and* $a = 0.056$. *Describe and interpret the results.*

Question 4.1.3 *Run the simulation again with the same parameters, but this time assign* a *to be* `-math.log(0.5) / paraPop`. *(This is* $a = -\ln 0.5/12 \approx 0.058$, *just slightly above the original value of* a.*) What do you observe?*

Question 4.1.4 *Run the simulation again with the same parameters, but this time assign* $a = 0.06$. *What do you observe?*

Question 4.1.5 *Based on these three simulations, what can you say about this model and its sensitivity to the value of* a*?*

Part 2: Constrained growth

An updated Nicholson-Bailey model incorporates a *carrying capacity* that keeps the host population under control. The carrying capacity of an ecosystem is the maximum number of organisms that the ecosystem can support at any particular time. If the population size exceeds the carrying capacity, there are not enough resources to support the entire population, so some individuals do not survive.

In this revised model, $P(t)$ is the same, but $H(t)$ is modified to be

$$H(t) = H(t-1) \cdot e^{-aP(t-1)} \cdot e^{r(1-H(t-1)/K)}$$

where K is the carrying capacity. In this new difference equation, the average number of surviving host offspring, formerly r, is now represented by

$$e^{r(1-H(t-1)/K)}.$$

Notice that, when the number of hosts $H(t-1)$ equals the carrying capacity K, the exponent equals zero. So the number of surviving host offspring is $e^0 = 1$. In general, as the number of hosts $H(t-1)$ gets closer to the carrying capacity K, the exponent gets smaller and the value of the expression above gets closer to 1. At the other extreme, when $H(t-1)$ is close to 0, the expression is close to e^r. So, overall, the number of surviving offspring varies between 1 and e^r, depending on how close $H(t-1)$ comes to the carrying capacity.

Write a function

```
NB_CC(hostPop, paraPop, r, c, a, K, years)
```

that implements this modified model and generates the same plots as the previous function.

Question 4.1.6 *Call your* NB_CC *function to simulate initial populations of 24 hosts and 12 parasitoids for 35 years. Use values of* $r = 1.5$, $c = 1$, $a = 0.056$, *and* $K = 40$. *Describe and interpret the results.*

Question 4.1.7 *Run your simulation with all three values of* a *that we used in Part 1. How do these results differ from the prior simulation?*